SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

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DEPARTMENT OF MATHEMATICS

QUESTION BANK

I SEMESTER

M.E. Structural Engineering

1918103 – Advanced Mathematical Methods

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Prepared by

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SRM VALLIAMMAI ENGNIEERING COLLEGE

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DEPARTMENT OF MATHEMATICS

Year & Semester : I /I
Section : M.E. (Structural Engineering)
Subject Code : 1918103
Subject Name : **ADVANCED MATHEMATICAL METHODS**
Degree & Branch : M.E – Structural Engineering
Staff in charge : **Mr. N. SUNDARAKANNAN**

S.No	QUESTIONS	COMPETENCE	LEVEL
UNIT -1 LAPLACE TRANSFORM TECHNIQUES FOR PARTIAL DIFFERENTIAL EQUATIONS Laplace transform, Definitions, properties – Transform error function, Bessel's function, Dirac Delta function, Unit Step functions – Convolution theorem – Inverse Laplace Transform: Complex inversion formula – Solutions to partial differential equations: Heat equation, Wave equation.			
	PART A - 2 MARK QUESTIONS		
1.	Define Laplace Transform of a function	Remembering	BTL1
2.	State the existence of Laplace Transform	Analyzing	BTL4
3.	*If* $L(f(t) = F(s)$, then prove that $L[e^{at} f(t)] = F(s-a)$	Applying	BTL3
4.	Find $L[t\cos^2 t]$	Analyzing	BTL4
5.	Determine $L\{2t - 3e^{2t}\}$	Analyzing	BTL4
6.	Find $L[e^{-4t}\sin 3t]$	Analyzing	BTL4
7.	State Initial and Final value theorems for Laplace transform	Remembering	BTL1
8.	Find $L\left[\frac{e^{-bt} - e^{-at}}{t}\right]$	Analyzing	BTL4
9.	State and prove that change of scale property in Laplace transforms	Applying	BTL3
10.	Define Laplace transform of Error function	Remembering	BTL1
11.	Find Laplace transform of $t\,e^{-2t}\cos t$	Analyzing	BTL4
12.	Define Bessel's function .	Applying	BTL3
13.	Find $L^{-1}\left(\frac{s-5}{s^2+25}\right)$	Analyzing	BTL4
14.	Show that $\int_0^t J_0(u)J_0(t-u)du = \sin t$	Understand	BTL2
15.	Find the inverse Laplace transform of $\log\left(\frac{s+a}{s+b}\right)$	Analyzing	BTL4

16.	Define Laplace transform of unit step function and find its Laplace transform	Remembering	BTL1
17.	Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$	Applying	BTL3
18.	State Convolution theorem	Applying	BTL3
19.	Define Dirac- Delta function.	Applying	BTL3
20.	Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$	Analyzing	BTL4
PART B - 13 MARK QUESTIONS			
1	Find (i) $L[t\sin 5t\cos 2t]$ $(ii)\ L[t\,e^{-t}\cos 2t]$	Applying	BTL3
2	Find the Laplace Transform of $erf(t^{\frac{1}{2}})$	Analyzing	BTL4
3	(i) Find $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$ (ii) Find the Laplace Transform of $f(t)=\begin{cases}\frac{t}{b}, & for\ 0<t<b\\ \frac{2b-t}{b}, & for\ b<t<2b\end{cases}$ given that f(t+2b)=f(t) for t > 0	Evaluating	BTL5
4	Using Convolution theorem, find the inverse Laplace Transform of $\frac{s}{(s^2+9)(s^2+25)}$	Analyzing	BTL4
5	Find the Laplace transform of $(i)\ J_0(t)\ \ (ii)\ t\,J_0(t)$	Evaluating	BTL5
6	Using Convolution theorem, evaluating $L^{-1}\left(\frac{s}{(s+a)(s^2+1)}\right)$	Applying	BTL3
7	Using Laplace transform, solve the IBVP $PDE: u_t = 3u_{xx}$, $BCs: u(\frac{\pi}{2},t)=0,\ u_x(0,t)=0,$ $ICs:\ u(x,0)=30\cos 5x.$	Evaluating	BTL5
8	Using Laplace transform, solve the IBVP $u_{tt}=u_{xx}\ \ 0<x<l, t>0,\ u(x,0)=0\ , u_t(x,0)=\sin(\frac{\pi x}{l})\ ,0<x<l,$ $u(0,t)=0\ and\ u(l,t)=0, t>0$	Evaluating	BTL5
9	Using Laplace transform, solve the IBVP $PDE: u_{tt}=u_{xx}\ \ 0<x<1, t>0,\ \ BCs: u(x,0)=u(1,t)=0, t>0$ $ICs:\ u(x,0)=\sin\pi x,\ u_t(x,0)=-\sin\pi x,\ 0<x<1$	Evaluating	BTL5
10	Using Laplace transform solve the following IBVP $PDE: ku_t = u_{xx}\ \ ,0<x<l, 0<t<\infty,$ $BCS: u(0,t)=0\ , u(l,t)=g(t)\ , 0<t<\infty,$ $IC:\ u(x,0)=0, 0<x<l.$	Analyzing	BTL4

11	Using Laplace transform method solve the initial boundary value problem described by $u_{tt} = c^2 u_{xx}$, $0 \le x < \infty$, $t \ge 0$, *subject to the conditions* $u(0,t) = 0$, $u(x,0) = 0$, $u_t(0,t) = l$ *for* $0 \le x < \infty$ *and* u *is bouded as* $x \to \infty$.	Evaluating	BTL5
12	Applying the convolution theorem to evaluating $L^{-1}\left[\frac{s}{(s^2+a^2)^2};t\right]$	Analyzing	BTL4
13	Derive the Laplace transform of Bessel functions (*i*) $J_0(x)$ (*ii*) $J_1(x)$ (ii) Find $L^{-1}\log\left(\frac{s^2-1}{s^2}\right)$	Analyzing	BTL4
14	Find the inverse Laplace Transform of $\frac{\sinh(x\sqrt{s})}{\sinh(l\sqrt{s})}$ in $0 < x < l$.	Evaluating	BTL5
Part C (15 Marks)			
1.	Verify Initial Value Theorem and Final Value Theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$	Evaluating	BTL5
2.	A infinitely long string having one end at x=0 is initially at rest along x-axis. The end x=0 is given transverse displacement f(t), when t >0. Find the displacement of any point of the string at any time.	Evaluating	BTL5
3.	Find the inverse Laplace Transform of $\frac{1}{(s+1)(s-2)^2}$ by complex inverse formula.	Analyzing	BTL4
4.	Solve the following IBVP using Laplace Transform PDE: $u_t = u_{xx}$, $0 < x < 1, t > 0$: BCs: $u(0,t) = 1$, $u(1,t) = 1$, $t > 0$ ICs : $u(x,0) = 1 + \sin \pi x$, $0 < x < 1$.	Applying	BTL3
UNIT -2 FOURIER TRANSFORM TECHNIQUES FOR PARTIAL DIFFERENTIAL EQUATIONS Fourier transform: Definitions, properties – Transform of elementary functions, Dirac Delta function – Convolution theorem – Parseval's identity – Solutions to partial differential equations: Heat equation, Wave equation, Laplace and Poison's equations.			
PART A - 2 MARK QUESTIONS			
1	Define Fourier transform of a function	Remembering	BTL1
2	Find the Fourier transform of $f(x) = \begin{cases} \cos x, \ in \ 0 < x < \pi \\ 0, \ \ otherwise \end{cases}$	Remembering	BTL1
3	State Fourier transform pairs	Analyzing	BTL4
4	State and prove shifting property in Fourier transform	Applying	BTL3
5	Find the Fourier transform of $e^{-a\|x\|}$,$-\infty < x < \infty$	Remembering	BTL1

6	Find the Fourier Transform of $f(x)=\begin{cases}1, in\ \lvert x\rvert<a\\ 0, in\ \lvert x\rvert>a\end{cases}$	Analyzing	BTL4
7	State and prove modulation property in Fourier transform	Applying	BTL3
8	State and prove change of scale property in Fourier transform	Applying	BTL3
9	State Fourier Cosine transform pair	Remembering	BTL1
10	State Fourier Sine transform pair	Remembering	BTL1
11	Find the Fourier Sine transform of e^{-ax} $,0<x<\infty$	Analyzing	BTL4
12	Find the Fourier Sine transform of 1/x	Analyzing	BTL4
13	Find the Fourier cosine transform of e^{-x}, $0<x<\infty$	Remembering	BTL1
14	Determine the frequency spectrum of the signal $g(t)=f(t)\cos\omega_c t$	Applying	BTL3
15	If F(α) is the Fourier Transform of f(x), then the Fourier Transform of f(ax) is $\frac{1}{\lvert a\rvert}F(\alpha/a)$	Applying	BTL3
16	Find the Fourier transform of Dirac Delta function	Analyzing	BTL4
17	State convolution Theorem on Fourier Transform	Remembering	BTL1
18	State Parseval's identity on Fourier Transform	Remembering	BTL1
19	Find the Fourier cosine Transform of $f(x)=\begin{cases}x\ ,\ 0<x<a\\ 0\ ,\ x>a\end{cases}$	Analyzing	BTL4
20	If F(s) is the Fourier transform of *f*(x), then find the Fourier transform of *f*(x) cos ax	Evaluating	BTL5
PART B – 13 MARK QUESTIONS			
1	Find the Fourier Transform of $f(x)=\begin{cases}1-\lvert x\rvert, if\ \lvert x\rvert<1\\ 0, if\ \lvert x\rvert>1\end{cases}$ hence deduce that $\int_0^\infty(\frac{\sin t}{t})^4dt=\frac{\pi}{3}$	Analyzing	BTL4
2	Find the Fourier Cosine and Sine Transform of e^{-2x} and evaluate the integrals (i) $\int_0^\infty\frac{\cos\alpha x}{\alpha^2+4}d\alpha$ $(ii)\int_0^\infty\frac{\alpha\sin\alpha x}{\alpha^2+4}d\alpha$	Analyzing	BTL4
3	Find the Fourier Transform of *f*(*x*) defined by $f(x)=\begin{cases}1-x^2, if\ \lvert x\rvert<1\\ 0, if\ \lvert x\rvert>1\end{cases}$. hence evaluate $\int_0^\infty\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$.	Analyzing	BTL4
4	Find the Fourier Transform of $f(x)=\begin{cases}1, if\ \lvert x\rvert<a\\ 0, if\ \lvert x\rvert>a\end{cases}$ hence deduce that $\int_0^\infty\left[\frac{\sin t}{t}\right]dt=\frac{\pi}{2}$ and $\int_0^\infty\left[\frac{\sin t}{t}\right]^2dt=\frac{\pi}{2}$	Evaluating	BTL5

5	(i) Using Fourier Sine Transform, prove that $\int_0^\infty \frac{x^2 dx}{(x^2+9)(x^2+16)} = \frac{\pi}{14}$ (ii) Find the Fourier transform of $e^{-a\|x\|}$ $if\ a>0$, deduce that $\int_0^\infty \frac{dx}{(x^2+a^2)} = \frac{\pi}{4a^3}, if\ a>0$	Evaluating	BTL5
6	A one-dimensional infinite solid, $-\infty < x < \infty$,is initially at temperature F(x). For times t>0, heat is generated within the solid at a rate of g(x ,t) units, Determines the temperature in the solid for t>o	Evaluating	BTL5
7	A uniform string of length L is stretched tightly between two fixed points at x=0 and x=L. If it is displaced a small distance $\in$ at a point x=b, 0<b<L, and released from rest at time t=0, find an expression for the displacement at subsequent times.	Evaluating	BTL5
8	Solve the heat conduction problem described by $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ $,0<x<\infty, t>0, \quad u(0,t)=u_0, t>0$ $u(x,0)=0\, 0<x<\infty, u\ and\ \frac{\partial u}{\partial x} both\ tend\ to\ zero\ as\ x \to \infty$	Analyzing	BLT4
9	Find the steady state temperature distribution u(x,y) in a long square bar of side π with one face maintained at constant temperature u_0 and other faces at zero temperature.	Analyzing	BLT4
10	Using the method of integral transform, solve the following potential problem in the semi-infinite Strip described by $PDE: u_{xx}+u_{yy}=0\ , 0<x<\infty\ ,0<y<a$ Subject to BCs: u(x,0)=f(x) , u(x,a)=0, u(x,y)=0 , $0<y<a\ , 0<x<\infty\ and\ \frac{\partial u}{\partial x}\ tend\ to\ zero\ as\ x \to \infty$	Analyzing	BTL4
11	Using the finite Fourier transform, solve the BVP described by PDE: $\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, 0<x<6\ , t>0$, subject to BC: $V_x(0,t)=0=V_x(6,t)$, IC :V(x,0)=2x.	Analyzing	BTL4
12	Solve the heat conduction problem described	Analyzing	BTL4

	$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$ $u(0,t) = u_0$ $t > 0$, $u(x,0) = 0$, $0 < x < \infty$, u *and* $\frac{\partial u}{\partial x}$ *both tend to zero* *as* $x \to \infty$		
13	Compute the displacement u(x ,t) of an infinite string using the method of Fourier transform given that the string is initially at rest and that the initial displacement is f(x), $-\infty < x < \infty$.	Creating	BTL6
14	Solve the following boundary value problem in the half-plane y > 0 described by PDE : $u_{xx} + u_{yy} = 0$, $-\infty < x < \infty$, $y > 0$ BCs : $u(x,0) = f(x)$, $-\infty < x < \infty$ u is bounded as $y \to \infty$; u *and* $\frac{\partial u}{\partial x}$ *both vanish as* $\|x\| \to \infty$	Evaluating	BTL5
Part C - 15 Marks			
1.	Using the Fourier Cosine Transform of e^{-ax} & e^{-bx} , show that $\int_0^\infty \frac{d\alpha}{(a^2+\alpha^2)(b^2+\alpha^2)} = \frac{\pi}{2ab(a+b)}$. $a>0$, $b>0$	Evaluating	BTL5
2.	Determine the temperature distribution in the semi-infinite medium $x \geq 0$. When the end x = 0 is maintained at zero temperature and the initial temperature distribution is *f*(*x*).	Creating	BTL6
3.	Obtain the solution of free vibration of a semi infinite string governed by PDE $u_{tt} = c^2 u_{xx}$ $0 < x < \infty$, $c > 0$ ICs: $u(x,0) = f(x)$, $u_t(x,0) = g(x)$, $0 < x < \infty$.	Creating	BTL6
4.	Solve the following Neumann Problem PDE: $u_{xx}(x,y) + u_{yy}(x,y) = 0, -\infty < x < \infty$ BC : $u_y(x,0) = f(x)$, $-\infty < x < \infty$ u and $\frac{\partial u}{\partial x}$ both vanish as$\|x\| \to \infty$.	Creating	BTL6

UNIT -3 CALCULUS OF VARIATIONS

Concept of variation and its properties – Euler's equation – Functional dependant on first and higher order derivatives – Functionals dependant on functions of several independent variables – Variational problems with moving boundaries – Problems with constraints – Direct methods – Ritz and Kantorovich methods.

PART A - 2 MARK QUESTIONS			
1	Define a functional	Remembering	BTL1
2	Write Euler's equation for functional.	Applying	BTL3

3	Write Euler-Poisson equation.	Remembering	BTL1
4	Find the exteremals of the functional $\int^{\pi}(y^2 - y'^2 - 2y\sin x)\,dx.$	Remembering	BTL1
5	Test for an extremum the functional Find the extremals of $\int_{x_0}^{x_1}(y^2 + y^{1^2} - 2ye^x)\,dx$	Applying	BTL3
6	Prove that the commutative character of the operators of variation and differentiation.	Applying	BTL3
7	Show that the symbols $\frac{d}{dx}$ *and* δ *commutative*	Remembering	BTL1
8	State Brachistochrone problem.	Remembering	BTL1
9	Find the curves on which the functional $\int_0^1 (y'^2 + 12xy)\,dx$ *with* $y(0) = 0$ *and* $y(1) = 1$ can be extremised.	Applying	BTL3
10	Write the ostrogradsky equation for the functional $I[z(x,y)] = \iint_D \left\{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + zf(x,y)\right\}dxdy$	Analyzing	BTL4
11	Write the ostrogradsky equation for the functional $\iint_D \left(\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2\right)dxdy$	Analyzing	BTL4
12	Define isoperimetric problems.	Applying	BTL3
13	Define moving boundaries.	Remembering	BTL1
14	Find the transversality condition for the functional $v = \int_{x_0}^{x_1} A(x,y).\sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\,dx$ with the right boundary moving along $y_1 = \varphi(x_1)$	Applying	BTL3
15	Test for an extremum the functional $I[y(x)\} = \int_0^1 (xy + y^2 - 2y^2y')\,dx$ y(0)=1, y(1)=2.	Evaluating	BTL5
16	Write down biharmonic equation.	Applying	BTL3
17	State Hamilton's Principle	Remembering	BTL1
18	Find the fundmental lemma of calculus of variation.	Remembering	BTL1
19	Explain direct methods in variational problem	Understand	BTL2
20	Explain Rayleigh-Ritz method.	Understand	BTL2
PART B - 13 MARK QUESTIONS			
1	(i) $I[y(x)\} = \int_0^1 (xy + y^2 - 2y^2y')\,dx$, $y(0) = 1$, $y(1) = 2.$ (ii) Find the extremals of $\int_{x_0}^{x_1}(y^{1^2} + 2yy^1 \;\; - 16y^2)\,dx$	Analyzing	BTL4
2	(i)Solve the extremal $v[y(x)] = \int_{x_0}^{x_1}\frac{dy}{dx}\left(1 + x^2\frac{dy}{dx}\right)dx.$ (ii)Find the extremal of the function	Applying	BTL3

	$v[y(x),z(x)]\int_{x_0}^{x}\left[2yz-2y^2+y'^2-z'^2\right]dx$.		
3	(i)Show that the straight line is the sharpest distance between two points in a plane. (ii) A curve c joining the points (x_1,y_1) and (x_2,y_2) is revolved about the x-axis. Find the shape of the curve, so that the surface area generated is a minimum.	Analyzing	BTL4
4	(i)State and Prove Brachistochrone problem .(ii) Show that the curve which extremize $I=\int_0^{\frac{\pi}{2}}[(y^{11})^2-y^2+x^2]dx$ given that $y(0)=0, y^1(0)=1, y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}, y^1\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ $is\ y=\sin x$.	Analyzing	BTL4
5	Find the shortest distance between the point (-1,5) and the parabola $y^2=x$	Analyzing	BTL4
6	(i)Find the plane curve of a fixed perimeter and maximum area. (ii)Find the extremals of the functional $\int_0^1(1+y''^2)\,dx$. Subject to $y(0)=0$, $y'(0)=1$, $y(1)=1$, $y'(1)=1$.	Evaluating	BTL5
7	Find the extremal of the functional $\int_0^{\pi}(y'^2-y^2+4y\cos x)\,dx.$ $y(0)=0$, $y(\pi)=0.$ Find the extremal of the functional $I(y,z)=\int_0^1(y'^2+z'^2+2y)\,dx;\ y(0)=1,\ y(1)=\frac{3}{2},$ $z(0)=0, z(1)=1)$	Evaluating	BTL5
8	Solve the boundary value problem $y^{11}-y+x=0\ (0\le x\le 1)\ y(0)=y(1)=0$ by Rayleigh Ritz method.	Applying	BTL3
9	To determine the shape of a solid of revolution moving in a flow of gas with least resistance.	Analyzing	BTL4
10	Derive the Euler's equation and use it, find the extremals of the functional $V[y(x)\}=\int_{x_0}^{x_1}(y^2+y'^2-2y\sin x)\,dx.$	Creating	BTL6
11	Find an approximate solution to the problem of the minimum, of the functional $J(y)=\int_0^1(y'^2-y^2+2xy)\,dx.$ y(0)=0=y(1) by Ritz method and compare it with the exact solution.	Evaluating	BTL5
12	Find the solid of maximum volume formed by the revolution of a given surface area.	Evaluating	BTL5
13	Find the extremals of the functional $v[y(x),z(x)\}=\int_0^{\pi/2}(y'^2+z'^2+2yz)\,dx.$ Given that	Evaluating	BTL5

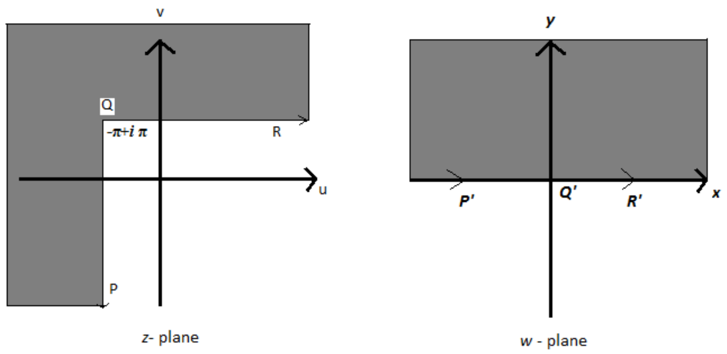
	$y(0)=0$, $y(\pi/2)=-1$, $z(0)=0$, $z(\pi/2)=1$		
14	Find the curve connect in the points A and B which is traversed by a particle sliding from A to B in the shortest time.	Creating	BTL6
	Part C – 15 Marks		
1.	Discuss the Branchistochrone Problem.	Remembering	BTL1
2.	Deterine the external of the functional $I(y(x)) = \int_{-a}^{a}\left(\frac{1}{2}\mu y''^2 + \rho y\right)dx$ that satisfies the condition $y(-a)=0,\ y'(-a)=0,\ y(a)=0,\ y'(a)=0$	Applying	BTL3
3.	Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume.	Remembering	BTL1
4.	Using Ritz method, find the approximate solution of the problem of the minimum of the functional $I(y(x)) = \int_0^2 \left(y'^2 + y^2 + 2xy\right)dx$, $y(0)=y(2)=0$ and compare with the exact solution.	Applying	BTL3

UNIT- 4 CONFORMAL MAPPING AND APPLICATIONS

Introduction to conformal mappings and bilinear transformations – Schwarz Christoffel transformation – Transformation of boundaries in parametric form – Physical applications : Fluid flow and heat flow problems.

PART A - 2 MARK QUESTIONS

1	Define Conformal Mapping and give an example.	Remembering	BTL1
2	Define Isogonal trandformation	Remembering	BTL1
3	Find the stagnation points of the flow represented by the complex potential $\Omega(z) = z^2$	Remembering	BTL1
4	Define Mobius transformation	Remembering	BTL1
5	Define cross ratio of Z_1 , Z_2 , Z_3 , Z_4	Understand	BTL2
6	Define Schwarz-Christoffel transformation.	Applying	BTL3
7	Define invariant point of the transformation w = f(z)	Remembering	BTL1
8	Find the fixed points of the transformation $w = \frac{z-2}{z+4}$	Applying	BTL3
9	Find the critical point of the transformation $W^2 = (Z-\alpha)(Z-\beta)$	Analyzing	BTL4
10	Find the fixed points of $w = \frac{3z+1}{z-1}$	Analyzing	BTL4
11	Let C be a curve in the z-plane with parametric equations x=F(t), y=G(t).Show that the transformation z= F(ω)+i G(ω) maps the curve C on to the real axis of the w-plane.	Evaluating	BTL5
12	Discuss the transformation defined by w= coshz.	Remembering	BTL1
13	Define equipotential lines.	Remembering	BTL1

14	State the formula for Schwarz Christoffel transformation	Understand	BTL2
15	Define Heat flux of heat flow.	Remembering	BTL1
16	State the basic assumptions of fluid flow.	Remembering	BTL1
17	Define streamline	Remembering	BTL1
18	Show that for a closed polygon, the sum of exponents in Schwarz Christoffel transformation is -2.	Applying	BTL3
19	Define complex temperature of heat flow	Understand	BTL2
20	Define velocity potential, source and sink	Remembering	BTL1
	PART B - 13 MARK QUESTIONS		
1	(i) Prove that the bilinear transformation transforms circles of the z plane into circles of the w plane, where by circles we include circles of infinite radius which are straight lines (ii) Define Bilinear transformation and find the bilinear transformation which maps $z_1=-2, z_2=0, z_3=2$ on to the points $w_1=\infty, w_2=1/2, w_3=3/4$.	Understanding	BTL2
2	Find a transformation which maps a polygon in the w-plane to the unit circle in the z-plane. 	Applying	BTL3
3	(i) Find the function harmonic in the upper half of the z-plane, which takes the prescribed values on x- axis given by $G(x)=\begin{cases}1., & x>0\\ 0, & x<0\end{cases}$. (ii)Find a BLT $z_1=0,\ z_2=-i,\ z=-1$ $in\ to\ the\ points$ $w_1=1,\ w_2=1\ \ w_3=0$	Remembering	BTL-1
4	Find the bilinear transformation which maps the points z=1, i, -1 into the points w=2, i, -2.Find the fixed and critical points of the transformation.	Understand	BTL2
5	Prove that a harmonic function $\Phi(x,y)$ remains harmonic under the transformation w=f(z) where f(z) is analytic and $f'(z)\neq 0$.	Understand	BTL2
6	Find the transformation which maps the semi-infinite strip bounded by v=0, u=0 and v=b into the upper half of the z-plane.	Analyzing	BTL4

7	Find the complex potential due to a source at z=-a and a sink at z=a equal strengths k, Determine the equipotential lines and streamlines and represent graphically and find the speed of the fluid at any point.	Evaluating	BTL5
8	Discuss the motion of a fluid having complex potential $\Omega(z) = ik \log z \ \ where\ k > 0$	Applying	BTL3
9	The complex potential of a fluid flow is given by $\Omega(z) = V_0(z + \frac{a^2}{z})$ where V_0 and α are positive constants. (a) Obtain equations for the streamlines and equipotential lines, represent them graphically and interpret physically. (b) Show that we can interpret the flow as that around a circular obstacle of radius a. (c) Find the velocity at any point and determine its value far from the obstacle (d) Find the stagnation points.	Evaluating	BTL5
10	Fluid emanates at a constant rate from an infinite line source perpendicular to the z plane at z=0 (a) Show that the speed of the fluid at a distance r from the source is V=k/r, where k is a constant (b) Show that the complex potential is $\Omega(z) = k \ln z.$ (c) What modification should be made in (b) if the line source is at z=a? (d) What modification is made in (b) if the source is replaced by a sink in which fluid is disappearing at a constant rate?	Analyzing	BTL4
11	(i) Find a bilinear transformation which maps points z= 0, -i, -1 into w= i, 1, 0 respectively (ii) Find the complex potential for a fluid moving with constant speed V_0 in a direction making an angle δ with the positive x-axis. Also determine the velocity potential and stream function.	Applying	BTL3
12	Find the transformation which transforms the semi infinite strip bounded by v= 0, v= π and u=0 onto the upper half z-plane.	Analyzing	BTL4
13	Show that the transformation $w = \frac{3 - z}{z - 2}$ transforms circle with center(5/2, 0) and radius 1/2 in the z-plane into the imaginary axis in w-plane and the interior of the circle into the right half of the plane.	Applying	BTL3
14	Find the transformation that maps an infinite strip $0 \le v \le \pi$ of the w-plane onto the upper half of z-plane.	Analyzing	BTL4
	Part C- 15 Marks		
1.	Find the complex potential for a fluid moving with constant speed V_0 in a direction making an angle δ with the positive	Evaluating	BTL5

	x axis. Also (i) Determine the velocity potential and stream function (ii)Determine the equations for the streamlines and equipotential lines.		
2.	Discuss the Schwarz- Christoffel transformation.	Analyzing	BTL4
3.	Find the Bilinear transformation which maps the points z= 1, i, -1onto w= i, 0, -i. Hence find the image of |z|<1 and the invariant points of the transformation.	Applying	BTL3
4.	Discuss the different types of Fluid flow and Heat Flow.	Analyzing	BTL4

UNIT-5 TENSOR ANALYSIS

Summation convention – Contravariant and covaraiant vectors – Contraction of tensors – Innerproduct – Quotient law – Metric tensor – Chrirstoffel symbols – Covariant differentiation – Gradient, divergence and curl.

PART A - 2 MARK QUESTIONS:

1	Define Tensor and Summation convention	Remembering	BTL1
2	Define Kronecker delta function of tensor	Remembering	BTL1
3	Define Covariant and Contravariant vectors	Remembering	BTL1
4	Define contraction of a tensor.	Applying	BTL3
5	If A_i is a covariant and B^i is a contravariant vector, show that $A_i B^i$ is an invariant	Applying	BTL3
6	State the Quotient law of tensors.	Applying	BTL3
7	Define reciprocal tensors.	Remembering	BTL1
8	Prove tha $[pr,q]+[qr,p]=\frac{\partial g_{pq}}{\partial x^r}$.	Remembering	BTL1
9	Define Symmetric and Skew-symmetric tensors	Remembering	BTL1
10	Define divergence of a contra variant vector	Remembering	BTL1
11	Prove that Kronecker delta (δ) is a mixed tensor of order 2.	Analyzing	BTL4
12	Define Metric tensor and Conjugate tensor	Applying	BTL3
13	State the Christoffel symbol of first and second kind.	Understand	BTL2
14	If *f* is a function of '*n*' variables x^i then write the differential of *f* using summation convention.	Applying	BTL3
15	Write the terms contained in $S=a_{ij}x^i x^j$, taking n=3.	Applying	BTL3
16	If $(ds)^2=(dr)^2+r^2(d\theta)^2+r^2\sin^2\theta(d\phi)^2$,find the value of [22.1] and [13,3]	Creating	BTL6
17	Prove that $divA_i=\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}A^k)}{\partial x^k}$.	Analyzing	BTL4
18	Define Curl of a contra variant vector.	Applying	BTL3
19	Given A^i and B^j are contra variant vectors, show that $A^i\ B^j$ is contra variant tensor of rank 2	Applying	BTL3

20	Prove that the property of the Christoffel symbols $\Gamma_{ij,k} + \Gamma_{kj,i} = \frac{\partial g_{ik}}{\partial q^j}$	Analyzing	BTL4
PART B – 13 MARK QUESTIONS			
1	(i)In a contravariant vector has components $\frac{dx}{dt}, \frac{dy}{dt}$ in rectangular coordinates, show that they are $\frac{dr}{dt}$ *and* $\frac{d\theta}{dt}$ in polar coordinates. (ii) If A^i is a contravariant vector and B_j is a covariant vector, prove that $A_i B^j$ is a mixed tensor of order 2.	Evaluating	BTL5
2	(i) State and prove Quotient law of tensor. (ii) Find g *and* g^{ij} corresponding to the metric $(ds)^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$	Analyzing	BTL4
3	Find the components of the first and second fundamental tensor in spherical coordinates.	Creating	BTL6
4	Find the components of the metric tensor and the conjugate tensor in the cylindrical co-ordinates..	Applying	BTL3
5	(i)Define Christoffel's symbols. Show that they are not tensors (ii) Prove that the covariant derivative of g^{ij} is zero.	Analyzing	BTL4
6	A covariant tensor has components $x+y,\ xy,\ 2z-y^2$ in rectangular co-ordinates. Find its covariant components in cylindrical co-ordinates.	Evaluating	BTL5
7	Given the covariant components in rectangular co-ordinates $2x-z,\ x^2y,\ yz$. Find the covariant components in (i) Spherical polar co-ordinates (r,θ,φ) (ii) Cylindrical co-ordinates(ρ,φ,z)	Analyzing	BTL4
8	Show that the tensors $g_{ij},\ g^{ij}$ and the Kroneckor delta δ_j^i are constants with respect to covariant differentiation.	Applying	BTL3
9	(i) Find the g *and* g^{ij} for the metric $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3 + 0dx^3dx^1$ (ii) Prove that the covariant derivative of g^{ij} is zero.	Evaluating	BTL5
10	If $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\phi)^2$, find the value of (i) [22,1] and [13,3] (ii) $\begin{Bmatrix}1\\22\end{Bmatrix}$ *and* $\begin{Bmatrix}3\\13\end{Bmatrix}$	Evaluating	BTL5
11	Determine the metric tensors $g_{ij},\ g^{ij}$ in cylindrical co-ordinates.	Evaluating	BTL5
12	Prove that the covariant derivative of g^{ij} is zero.	Evaluating	BTL5
13	Prove that	Evaluating	BTL5

	(i) $\frac{\partial g_{ij}}{\partial x^k} = [ik, j] + [jk, i]$ (ii) $\frac{\partial g^{ij}}{\partial x^k} = -g^{jl}\begin{Bmatrix} i \\ lk \end{Bmatrix} - g^{im}\begin{Bmatrix} j \\ mk \end{Bmatrix}$		
14	Given the metric $ds^2 = (dx^1)^2 + (x^1)^2(dx^2)^2 + (x^1)^2 \sin^2(x^2)(dx^3)^2$ find the Christoffel symbols [22 ,1] , [32 , 3] $\begin{Bmatrix} 2 \\ 2 \quad 1 \end{Bmatrix}$ *and* $\begin{Bmatrix} 3 \\ 3 \quad 2 \end{Bmatrix}$	Evaluating	BTL5
15.	A covariant tensor has componets xy , $2y - z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates	Evaluating	BLT5
	Part C – 15 Marks		
1.	(i) Show that any innerpoduct of the tensor A_m^l and B_r^{pq} is a tensor of rank three. (ii) If g denotes the determinant of g_{ij} then prove that $\begin{Bmatrix} i \\ i \quad j \end{Bmatrix} = \frac{\partial}{\partial x^i}(\log \sqrt{g})$	Creating	BTL6
2.	Explain short notes about metric tensor with suitable example.	Analyzing	BTL4
3.	Discuss the Curl and Divergent of a tenor.	Analyzing	BTL4
4.	A covariant tensor has components xy, $2y-z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.	Applying	BTL3

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