7.2. R = (A, B, C, D, E). We decompose it into R_1 = (A, B, C), R_2 = (A, D, E). The set of functional dependencies is: $A \rightarrow BC$, $CD \rightarrow E$, $B \rightarrow D$, $E \rightarrow A$. Show that this decomposition is a lossless-join decomposition.

$R_1 \cap R_2 = A$; $(A \rightarrow BC) \Rightarrow (A \rightarrow ABC) \Rightarrow (R_1 \cap R_2 \rightarrow R_1) \Rightarrow$ this is a lossless-join decomposition.

7.16. The same R and F. R_1 = (A, B, C), R_2 = (C, D, E). Show that this decomposition is not a lossless-join decomposition.

r:

A	B	C	D	E
1	1	α	1	1
2	2	α	2	2

$\Pi_{A,B,C}(r)$

A	B	C
1	1	α
2	2	α

$\Pi_{C,D,E}(r)$

C	D	E
α	1	1
α	2	2

$\Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) \neq r$

A	B	C	D	E
1	1	α	1	1
1	1	α	2	2
2	2	α	1	1
2	2	α	2	2

7.18. The same R and F. R_1 = (A, B, C), R_2 = (A, D, E). Show that this decomposition is not a dependency-preserving decomposition.

$F_1 = \{A \rightarrow BC\}$ $F_2 = \{E \rightarrow A\}$

$(F_1 \cup F_2)^+ \neq F^+$

7.21. Give a lossless-join decomposition into BCNF of R = (A, B, C, D, E) with the set of functional dependencies: $A \rightarrow BC$, $CD \rightarrow E$, $B \rightarrow D$, $E \rightarrow A$.

result : = {R};

$F^+ = \{A \rightarrow ABCDE, B \rightarrow D, BC \rightarrow ABCDE, CD \rightarrow ABCDE, E \rightarrow ABCDE, \ldots\}$.

R is not in BCNF.
$B \rightarrow D$ is a non-trivial f.d. that holds on R, $B \cap D = \emptyset$, and $B \rightarrow ABCDE$ is not in F^+. Therefore,
result := (result – R) $\cup$ (R – D) $\cup$ (B, D), i.e. (A, B, C, E) $\cup$ (B, D).

(A, B, C, E) and (B,D) are in BCNF. So this is a decomposition of R into BCNF.

7.24. Give a lossless-join dependency-preserving decomposition into 3NF.

1) Construct a canonical cover of F. In our case $F_C = F$.
2) Initially we have an empty set of R_j (j = 0). Therefore, none of R_j contains ABC (we take a dependency from the canonical cover $A \rightarrow B$). So R_1 = (A, B, C). Consider $CD \rightarrow E$. CDE is not in R_1, hence we add R_2 = (C, D, E). Similarly, we add R_3 = (B, D), and R_4 = (E, A).
3) R_1 contains a candidate key for R, therefore we do not need to add a relation consisting of a candidate key.

Finally, the received decomposition is (A, B, C), (C, D, E), (B, D), (E, A).

I. Suppose we have a database for an investment firm, consisting of the following attributes:
B – Broker,
O – Office of a broker
I – Investor
S – Stock
Q – Quantity of stock owned by an investor
D – dividend paid by a stock.
Hence, the overall schema is R = (B, O, I, S, Q, D).
Assume that the following f.d. are required to hold on this d.b.
$I \rightarrow B$, $IS \rightarrow Q$, $B \rightarrow O$, $S \rightarrow D$.

1) List all the candidate keys for R.
2) Give a lossless-join decomposition of R into BCNF.
3) Give a lossless-join decomposition of R into 3NF preserving f.d. Is you answer is in BCNF?

1) ***I and S must be in any candidate key*** since they do not appear on the right of any f.d. The question is whether they form a complete candidate key. And yes, $IS \rightarrow ISDBOQ$. Hence, the only candidate key is IS.

2) 1. Decompose R by $I \rightarrow B$ into R_1 = (I, B), R_2 = (I, O, S, Q, D).
   2. R_1 is in BCNF.

3. Decompose R_2 by $S \to D$ into $R_{21} = (S, D)$, $R_{22} = (O, I, S, Q)$.
4. R_{21} is in BCNF.
5. Decompose R_{22} by $I \to O$ into $R_{221} = (I, O)$, $R_{222} = (I, S, Q)$.
6. R_{221} is in BCNF.
7. R_{222} is in BCNF.

The decomposition is (I, B), (S, D), (I, O), (I, S, Q).

An alternative answer is (I, B), (S, D), (B, O), (I, S, Q).

3) (I, B), (S, D), (B, O), (I, S, Q).
The answer is in BCNF.

II. Consider a relational schema R with attributes A, B, C, D, E and the set of functional dependencies $A \to CD$, $B \to CE$, $E \to B$.

1) Give a lossless-join decomposition of R into BCNF.
2) Give a lossless-join decomposition of R into 3NF preserving f.d. Is you answer is in BCNF?

1) 1. Decomposition by $A \to CD$. $R_1 = (A, B, E)$, $R_2 = (A, C, D)$.
2. Decomposition of R_1 by $E \to B$. $R_{11} = (A, E)$, $R_{12} = (B, E)$.
(A, E), (B, E) and (A, C, D) form a decomposition into BCNF.

2) 1. $A \to CD \quad \Rightarrow \quad R_1 = (A, C, D)$.
2. $B \to CE \quad \Rightarrow \quad R_2 = (B, C, E)$.
3. $E \to B$, but E, B are in R_2.
4. A candidate key is AB (or AE). It is neither in R_1 nor in R_2. Hence, we add $R_3 = (A, B)$.
The decomposition we got is (A, C, D), (B, C, E), (A, B).