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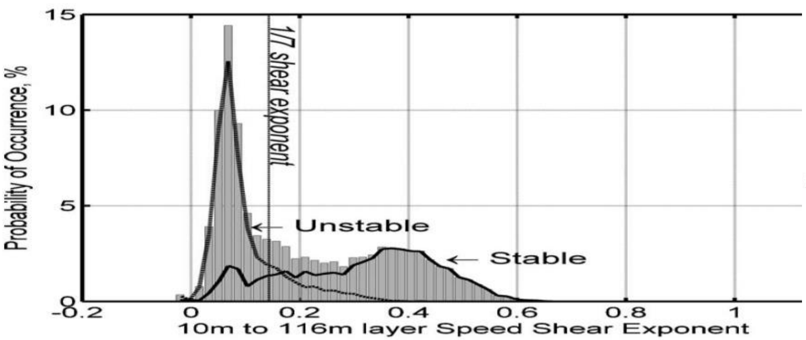
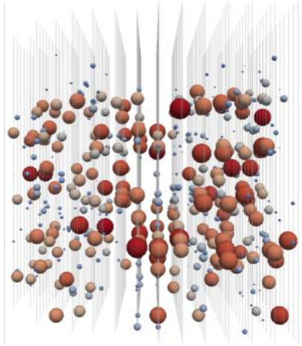
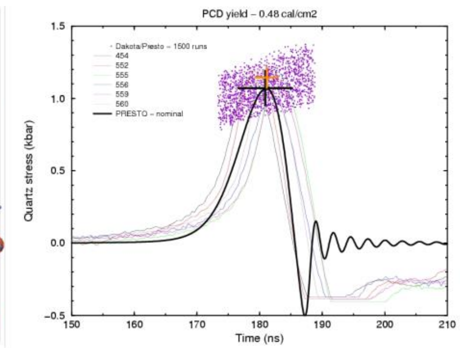


DAKOTA





Dakota Sensitivity Analysis and Uncertainty Quantification, with Examples

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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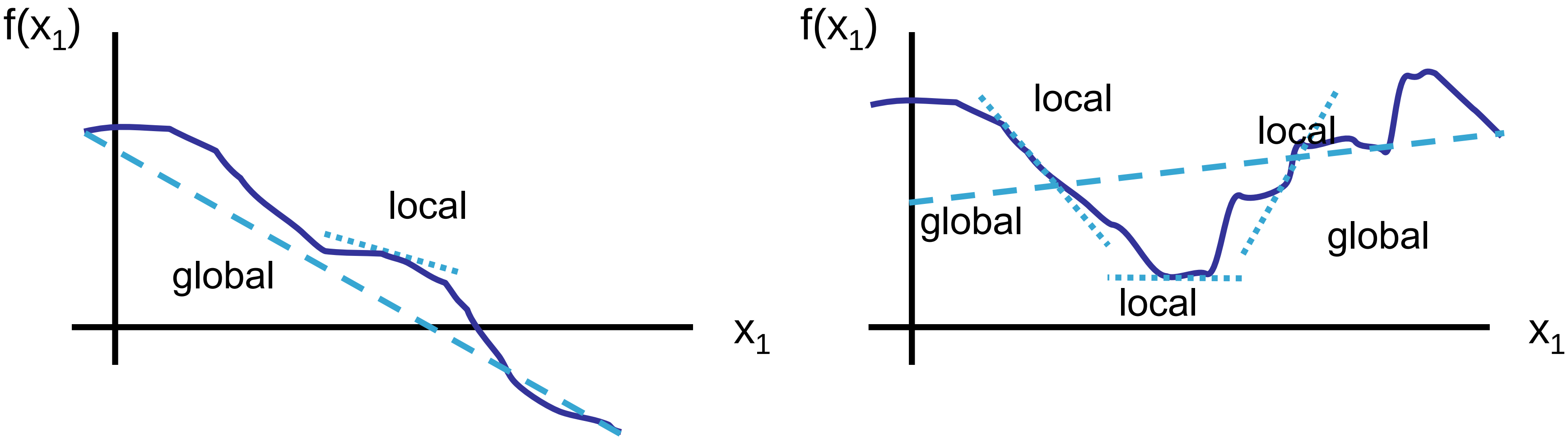
Dakota Sensitivity Analysis (SA)

- SA goals and examples
- Global SA approaches and metrics available in Dakota
- Select Dakota examples for parameter studies and global SA

Why Perform Sensitivity Analysis?

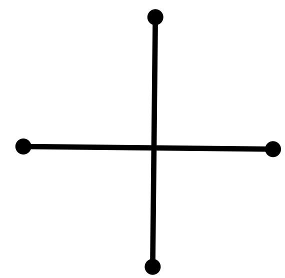
- What? Understand code output variations as input factors vary
- Why? Identify most important variables and their interactions
  - Identify key model characteristics: smoothness, nonlinear trends, robustness
  - Provide a focus for resources
    - Data gathering and model development
    - Code development
    - Uncertainty characterization
  - Screening: Identity the most important variables, down-select for further UQ or optimization analysis
  - Can have the side effect of identifying code and model issues
  - Data can be used to construct surrogate models
- Dakota SA formalizes and generalizes one-off sensitivity studies you're likely already doing
- Provides richer global sensitivity analysis methods

Sensitivity Analysis: Influence of Inputs on Outputs



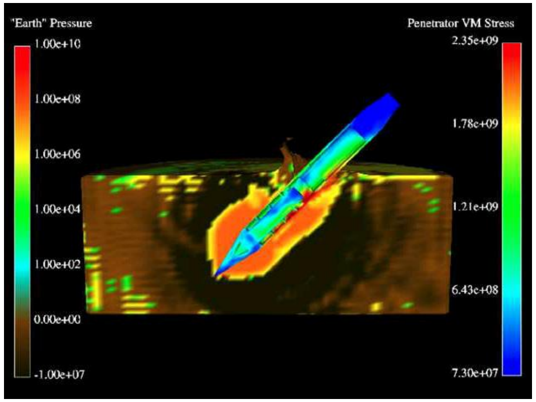
Assess variations in f(x1) due to (small or large) perturbations in x1.

- Local sensitivities
  - Partial derivatives at a specific point in input space.
  - Given a specific x1, what is the slope at that point?
  - Can be estimated with finite differences
- **Global sensitivities**
  - Found via sampling and regression.
  - What is the general trend of the function over all values of x1?
  - Typically consider inputs uniformly over their whole range



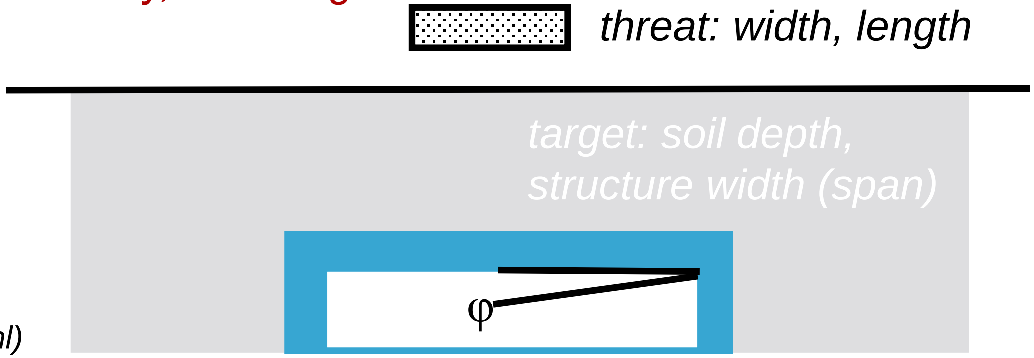
many already do basic SA; perturb from nominal, see effect

Global Sensitivity Analysis Example: Earth Penetrator

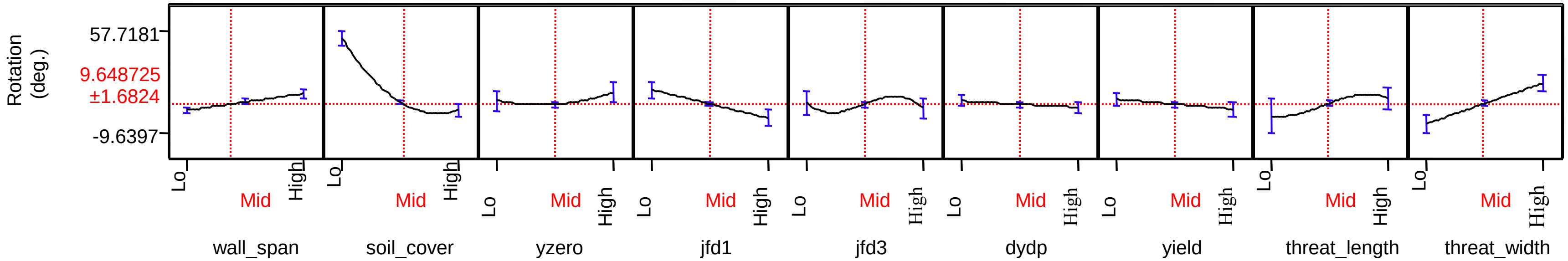


Notional model for illustration purposes only (http://www.sandia.gov/ASC/library/fullsize/penetrator.html)

12 parameters describing target & threat uncertainty, including…



- Underground target with external threat: assess sensitivity in target response to target construction and threat characteristics
- Response: angular rotation (ϕ) of target roof at mid-span
- Analysis: CTH Eulerian shock physics code; JMP stats
- Revealed most sensitive input parameters and nonlinear relationships



Global SA Example:
Nuclear Reactor Thermal-Hydraulics Model

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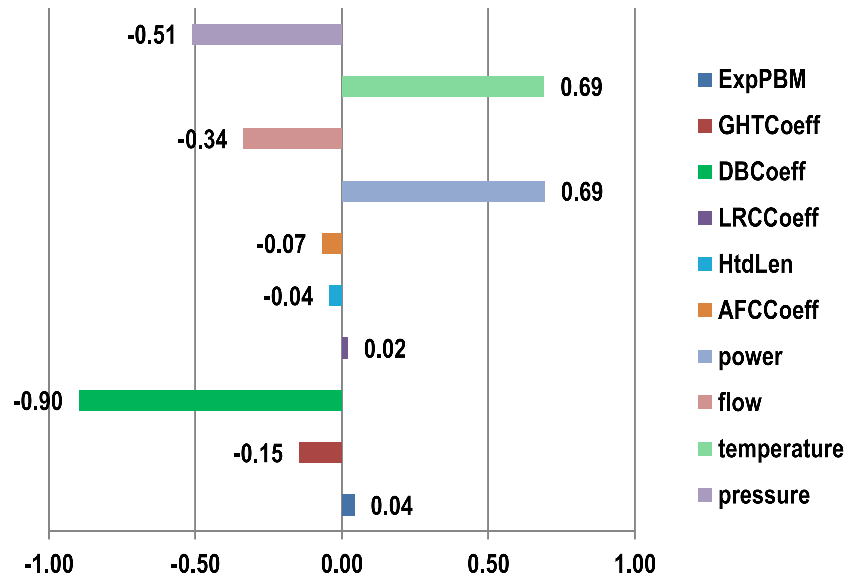
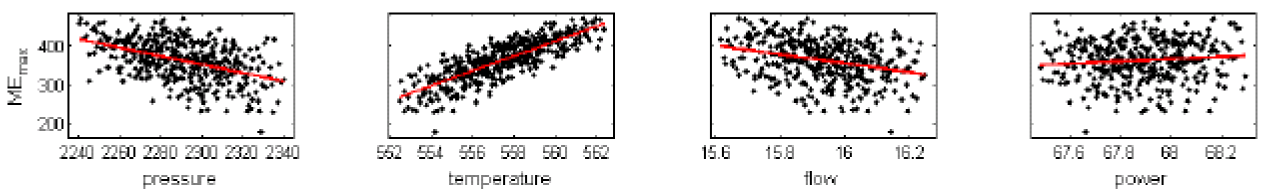
- Assess parameter influence on boiling rate, a key crud predictor
- Dakota correlation coefficients: strong influence of core operating parameters (pressure more important than previously thought)
- Dittus-Bolter correlation model may dominate model form sensitivities (also nonlinear effects of ExpPBM)
- Scatter plots help visualize trend in input/output relationships

parameter influence on number of boiling sites



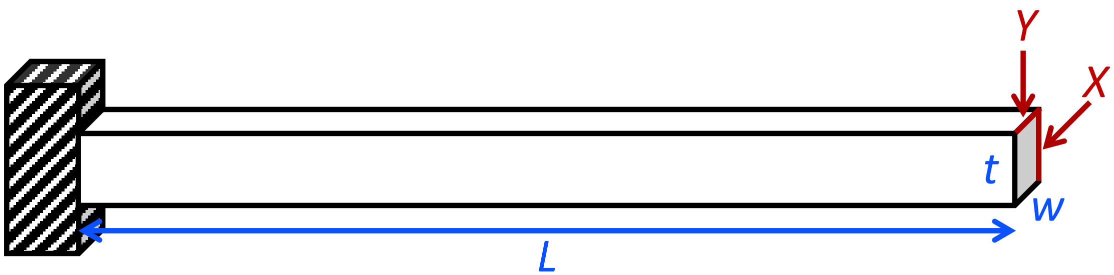


sensitivity of mass evaporation rate (max) to operating parameters

Group Discussion Questions: Your Sensitivity Analysis Practice

- Do you currently perform sensitivity analysis or parameter perturbations?
- What are example SA questions you (could) ask in your domain?
- How do (would) you answer them?
- What measures of sensitivity, ranking, or importance are you most familiar with?
- What are the key challenges you face?

Cantilever Beam Model



Parameters:

L: length (in)

w: width (in)

t: thickness (in.)

ρ: density (lb/ft^3)

E: Young's modulus (lb/in^2)

X: horizontal load (lb)

Y: vertical load (lb)

Responses:

M: mass (lb)

S: stress (lb/in^2)

D: displacement (in)

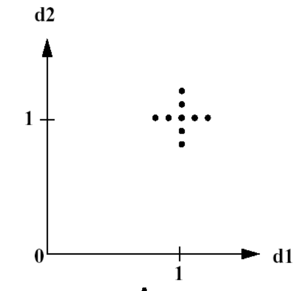
$$M = \rho * wt * \frac{L}{12^3}$$

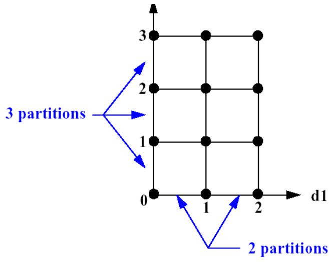
$$S = \frac{600}{wt^2} Y + \frac{600}{w^2 t} X$$

$$D = \frac{4L^3}{Ewt} \sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{X}{w^2}\right)^2}$$

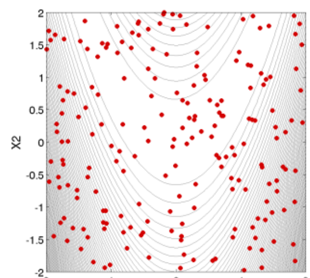
Global Sensitivity Analysis in Dakota

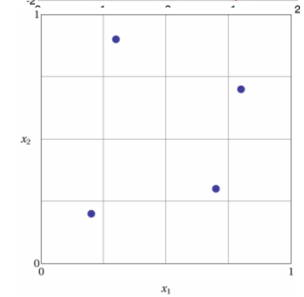
- Assess effect of input variables considered jointly over their whole range. Dakota process:
  - Specify variables: lower and upper bounds
  - Specify method: e.g., uniform random sampling
  - Specify responses: compute response value at each sample point
  - Run Dakota and analyze input/output relationships





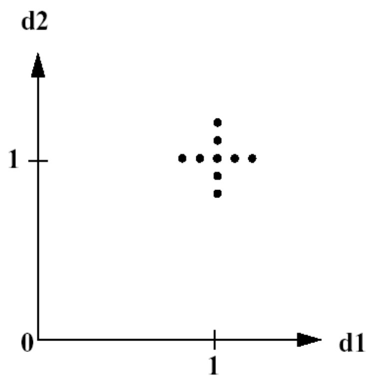
- Sample designs (methods) available:
  - Parameter studies: list, centered, grid, vector, user
  - Random sampling: Monte Carlo, Latin hypercube, Quasi-MC, CVT
  - DOE/DACE: Full-factorial, orthogonal arrays, Box-Behnken, CCD
  - Morris one-at-a-time
  - Sobol indices via variance-based decomposition, polynomial chaos



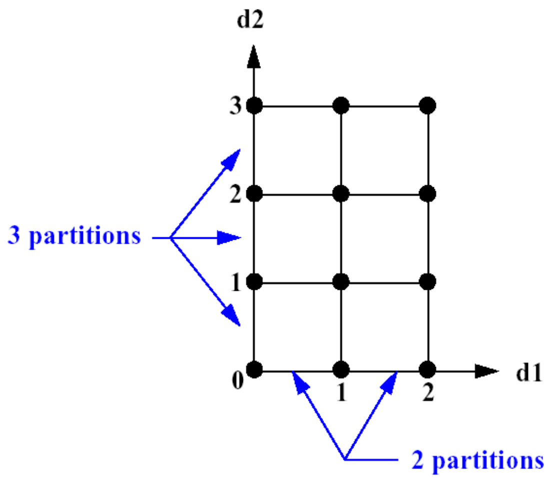


- Metrics: trends, correlations, main/interaction effects, Sobol indices, importance factors/local sensitivities

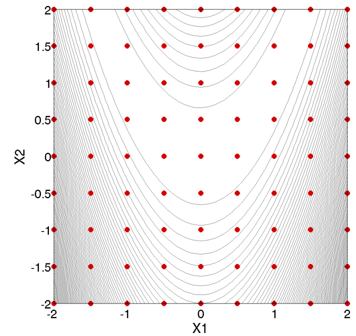
Basic Dakota SA for Cantilever: Centered and Grid Parameter Studies



- Start at nominal values, perturb up and down in each coordinate direction
- Specify the parameter variations, which responses to study
- See Dakota input and output (following slides)



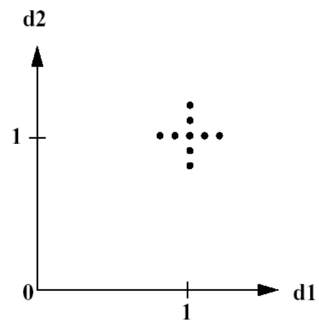
- What changes to Dakota input will instead perform the grid parameter study at left?
- Dakota Reference Manual helps with keyword choice…
- What are benefits/drawbacks of these methods?



Example: uniform grid over [-2.0, 2.0]

Dakota Input File: Cantilever Centered Parameter Study

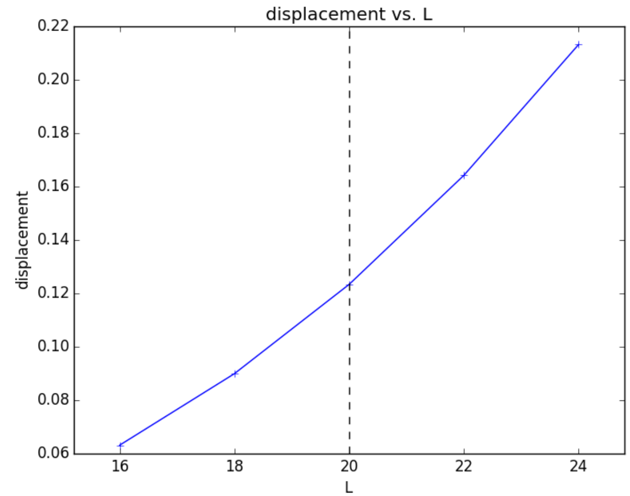
```
environment
  tabular_data output_precision 1e-16

method
  centered_parameter_study
    step_vector 0.1 0.1 2.0
                10 1.e5 5. 10.
    steps_per_variable 2

variables
  active all
  continuous_design = 3
    initial_point  1.0  1.0  20.0
    descriptors    'w'  't'  'L'
  continuous_state = 4
    initial_state  500.  29.E+6  50.  100.
    descriptors    'p'   'E'     'X'  'Y'

interface,
  fork
    analysis_driver = 'driver.sh'

responses,
  num_objective_functions = 3
    response_descriptors = 'mass' 'stress' 'displacement'
  no_gradients
  no_hessians
```

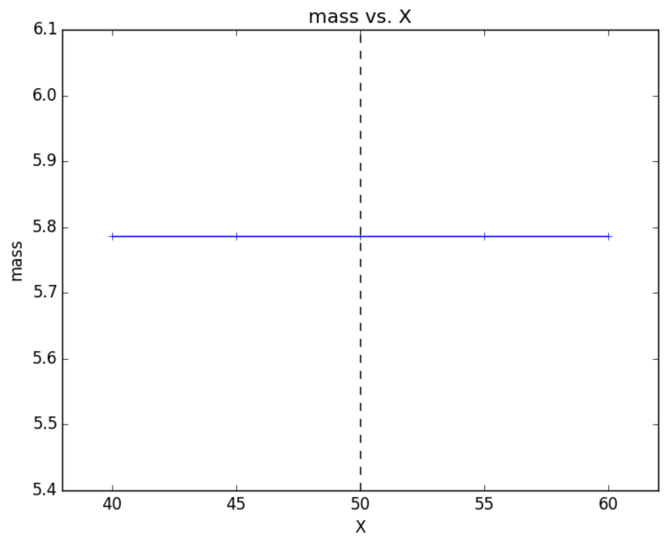


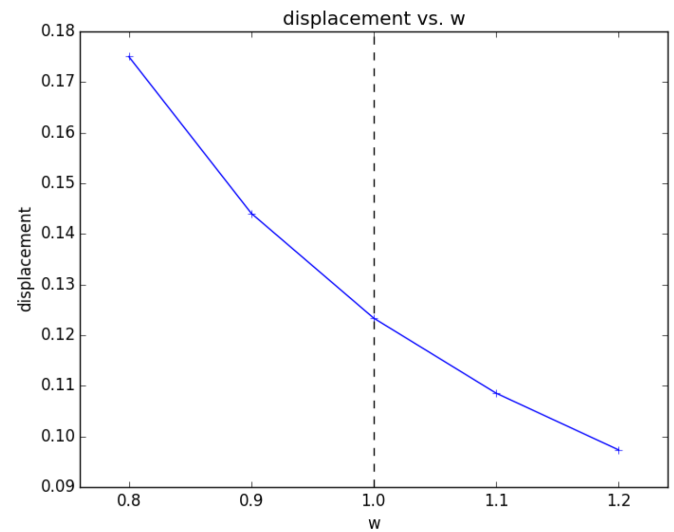
- Catalog variable/response sets to tabular file
- Algorithm configuration: steps in each variable
- Center point: initial point / initial state
- How parameters are mapped to responses
- Responses from simulation

Results: Centered Parameter Study

- Python plots of Dakota tabular file
- Univariate effects of parameters on each response
- What do you observe?
- What are benefits/drawbacks?

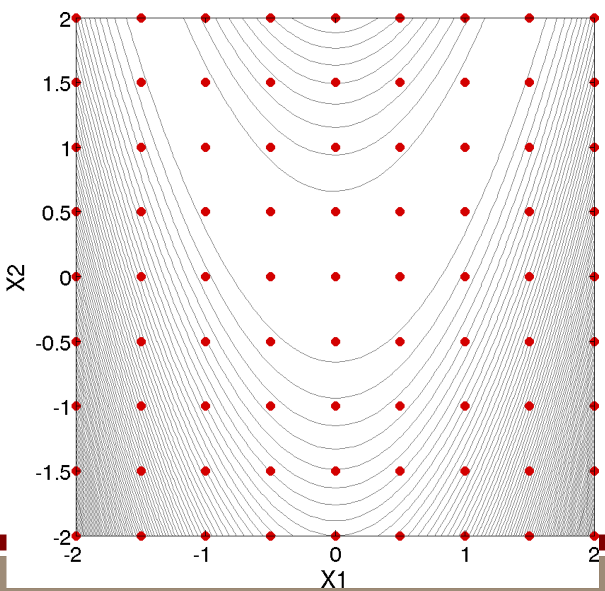






Exercise: Multi-dimensional Parameter Study

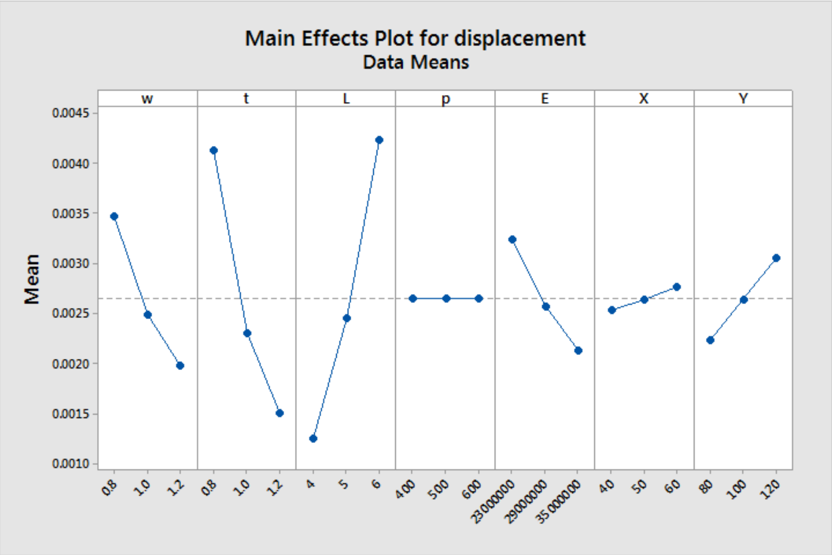
- Goal: understand how responses *area, stress, and displacement* vary with respect to the inputs *w* and *t* on a grid of points.
- *Exercise: change previous input file to run the mod_cantilever computational model at a grid of points over [1.0, 4.0] using the* `multidim_parameter_study` *method*
- Try 9 points in one dimension, 6 in the other
- See method and variable commands in Dakota reference manual
- What parts of the file did you have to change?



Example: uniform grid over [-2.0, 2.0]

Dakota Input File and Results: Cantilever Multi-dimensional Parameter Study

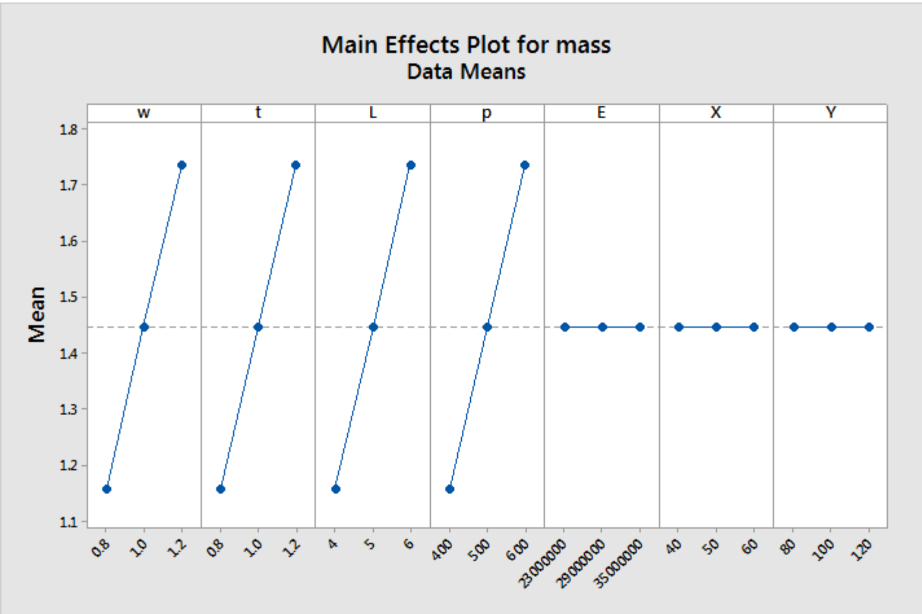
```
environment
  tabular_data output_precision 1e-16

method
  multidim_parameter_study
    partitions = 2 2 2 2 2 2 2

variables
  active all
  continuous_design = 3
    upper_bounds = 1.2 1.2 6.0
    lower_bounds = 0.8 0.8 4.0
    descriptors    "w"     "t"     "L"
  continuous_state = 4
    upper_bounds = 600. 35.E+6 60. 120.
    lower_bounds = 400. 23.E+6 40. 80.
    descriptors    'p'   'E'   'X'  'Y'

interface,
fork
  analysis_driver = 'driver.sh'

responses,
num_objective_functions = 3
    response_descriptors = 'mass' 'stress' 'displacement'
no_gradients
no_hessians
```

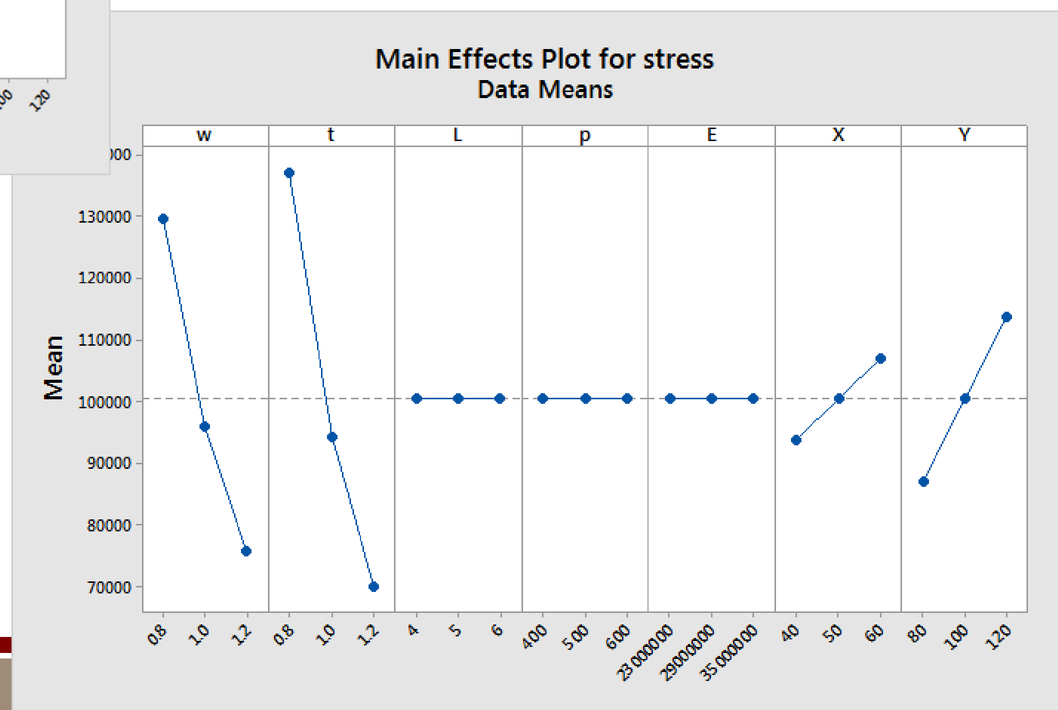


Dakota tabular data plotted with Minitab

What are benefits/drawbacks?

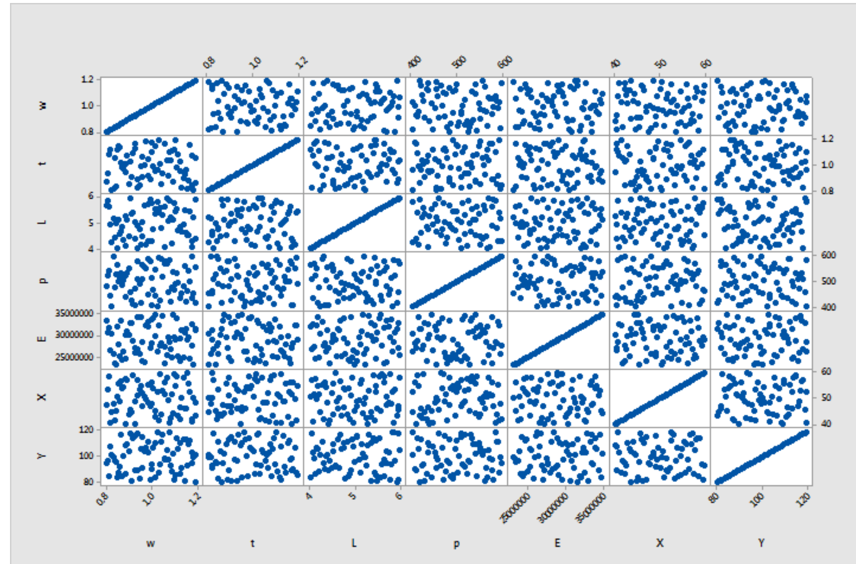
Dakota Input File and Results: Cantilever Multi-dimensional Parameter Study





Workhorse SA Method: Random Sampling

- Generate space filling design (typically Monte Carlo or Latin hypercube with samples = 2x or 10x number of variables)
- Run model at each point
- Analyze input/output relationships with
  - Correlation coefficients
    - Simple correlation: strength and direction of a linear relationship between variables
    - Partial correlation: like simple correlation but adjusts for the effects of the other variables
    - Rank correlations: simple and partial correlations performed on "rank" of data
  - Regression and resulting coefficients
  - Variance-based decomposition
  - Importance factors



Two-dimensional projections of LHD for Cantilever
(plotted with Minitab)

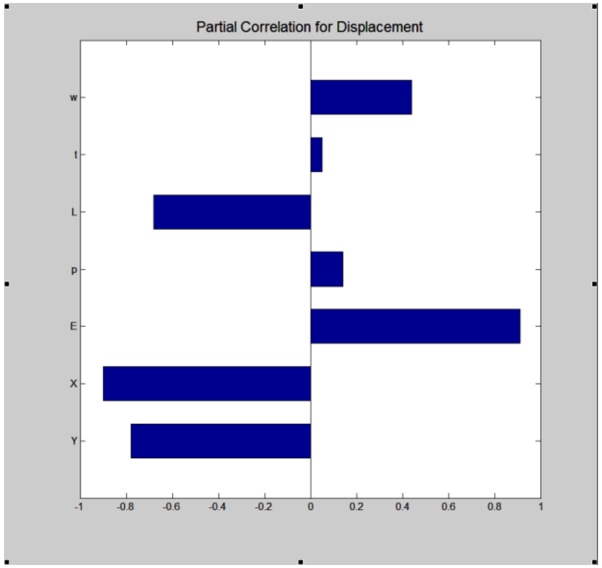
Dakota Input File: Cantilever LHS Study

```
method
  sampling
  sample_type lhs
  samples = 70
  seed = 3845

variables
  active all
  continuous_design = 3
    upper_bounds = 1.2 1.2 6.0
    lower_bounds = 0.8 0.8 4.0
    descriptors    "w"     "t"     "L"
  continuous_state = 4
    upper_bounds = 600. 35.E+6 60. 120.
    lower_bounds = 400. 23.E+6 40. 80.
    descriptors    'p'   'E'   'X'  'Y'

interface
  fork
    analysis_driver = 'driver.sh'

responses
  response_functions = 3
  descriptors = 'mass' 'stress''displacement'
  no_gradients no_hessians
```

Global Sampling Results for Cantilever

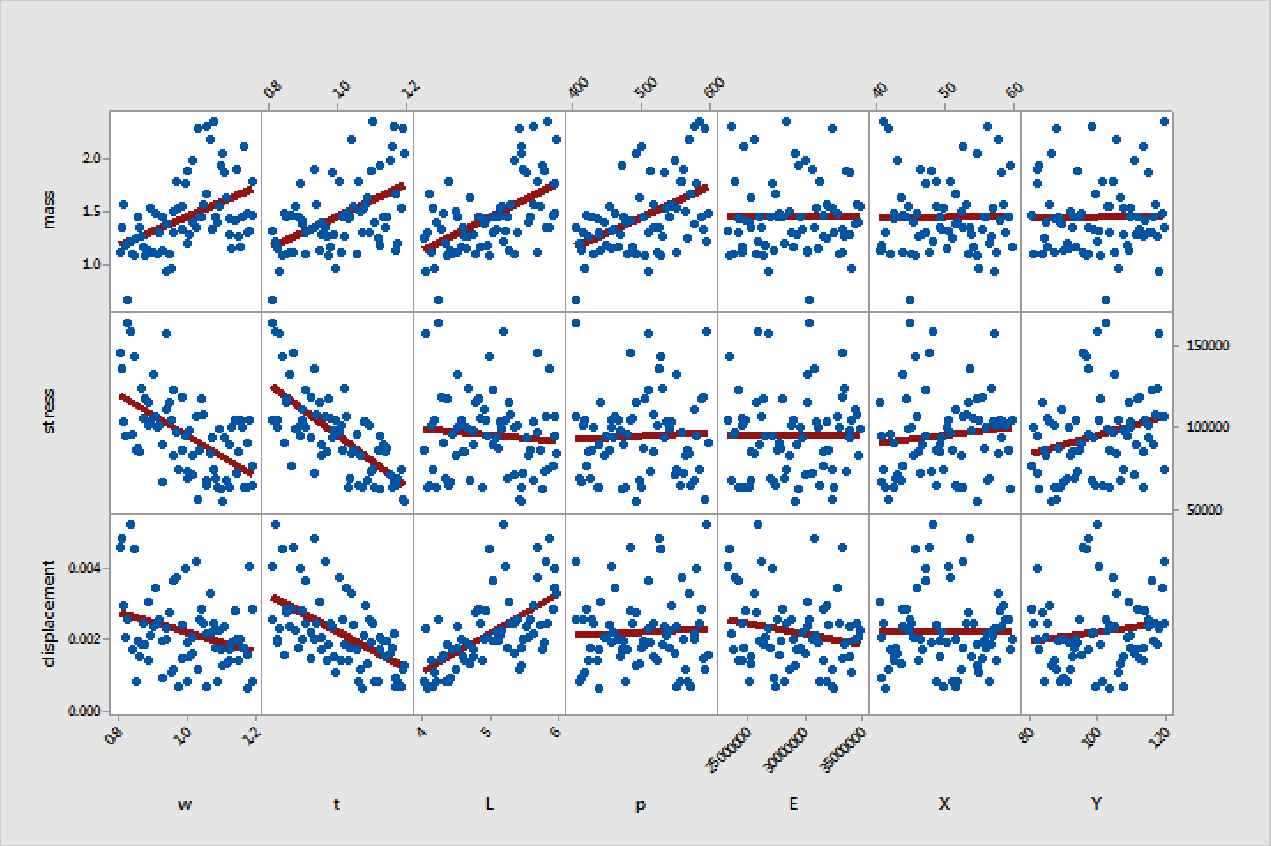
Partial Correlation Matrix for Cantilever			
	mass	stress	displacement
w	0.95	-0.96	-0.78
t	0.95	-0.97	-0.90
L	0.96	-0.17	0.91
p	0.95	0.11	0.14
E	-0.08	-0.13	-0.68
X	-0.03	0.54	0.05
Y	0.12	0.82	0.44

correlation coefficients from Dakota console output (colored w/ Excel)



(plotted with Matlab)

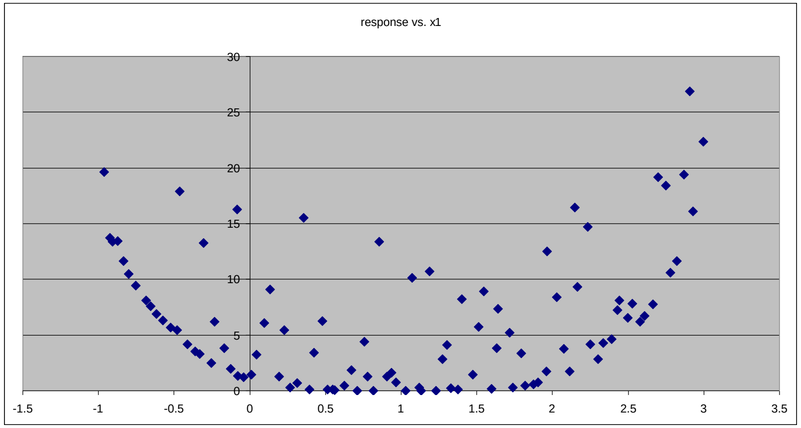
Dakota tabular data plotted in Minitab (can use Matlab, JMP, Excel, etc.)



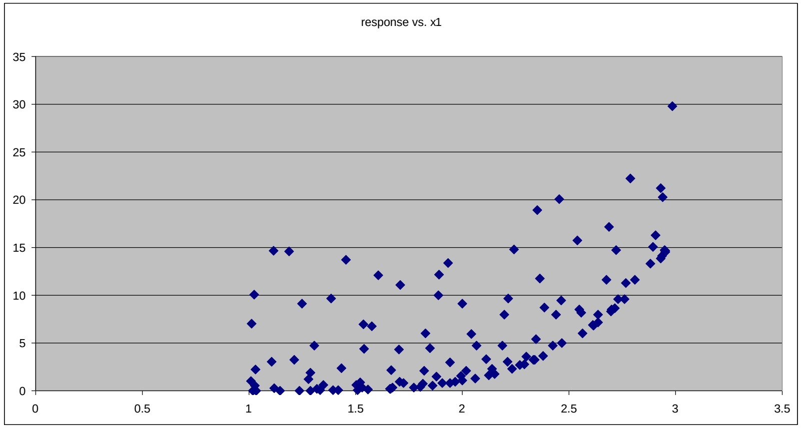
Group discussion

- What is expected, limited about this approach?
- What approaches would you take?
- What assumptions are we making? How would changing them affect results?

Bounds = [-1, 3]

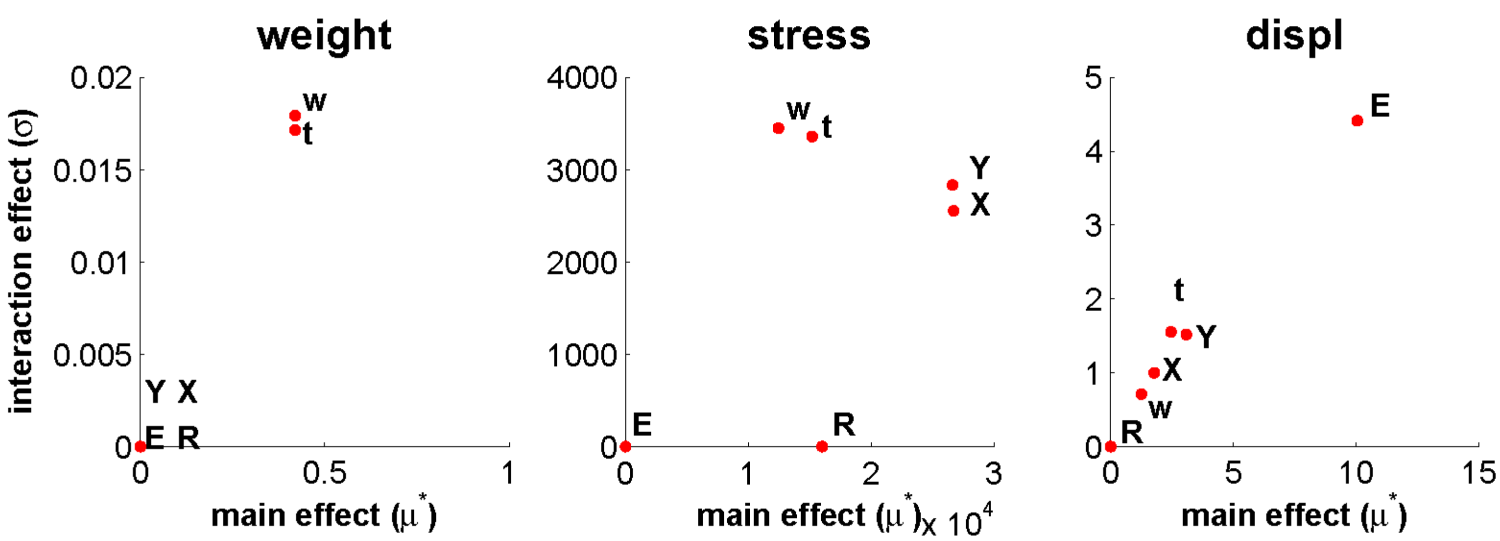


Bounds = [1, 3]



Morris One-at-a-Time (MOAT)

- Sample paths around global space in coordinate directions
- Give good measure of main (linear, first-order) and interaction / nonlinear effect for modest simulation budget
- How would you know how to configure Dakota to do this study?



Other SA Approaches Require Changing Method

- Dakota Reference Manual guides in specifying keywords

```
method,
sampling
    sample_type lhs
    seed =52983
    samples = 100
```

LHS Sampling

```
method,
    dace oas
    main_effects
    seed =52983
    samples = 500
```

Main Effects Analysis using Orthogonal Arrays

```
method,
sampling
    sample_type lhs
    seed =52983
    samples = 500
    variance_based_decomp
```

Variance-based Decomposition using LHS Sampling

```
method,
    psuade_moat
    partitions = 3
    seed =52983
    samples = 100
```

Morris One-At-a-Time

Dakota Sensitivity Analysis Summary

- What? Understand code output variations as input factors vary; main effects and key parameter interactions.
- Why? Identify most important variables and their interactions
- How? What Dakota methods are relevant? What results?

Category	Dakota method names	univariate trends	correlations	modified mean, s.d.	main effects Sobol inds.	importance factors / local sensis
Parameter studies	centered, vector, list	P				
	grid		D		P	
Sampling	sampling, dace lhs, dace random, fsu_quasi_mc, fsu_cvt with variance_based_decomp...	P	D		D	
DACE (DOE-like)	dace {oas, oa_lhs, box_behnken, central_composite}		D		D	
MOAT	psuade_moat			D		
PCE, SC	polynomial_chaos, stoch_collocation				D	D
Mean value	local_reliability					D

multi-purpose!

D: Dakota
P: Post-processing (3rd party tools)

- Also see Dakota Usage Guidelines in User's Manual

Common Question: UQ versus SA

What distinguishes sensitivity analysis from uncertainty analysis?

- With SA you primarily gain information about variables
  - Rank importance of parameters and characterize in what way they influence responses
  - Sometimes called inverse UQ
  - Secondarily, characterize model properties
- With UQ you primarily gain information about responses
  - Statistical properties of output responses
  - Intervals indicating bounds on response
  - Likelihood (probability of failure)

- Some methods can be used for both, e.g.,
  - LHS is often used for SA (correlations) and UQ (moments, PDFs, CDFs)
  - Polynomial chaos expansions (PCE) thought of as a UQ method, but also efficiently produce Sobol indices for ranking parameter influence

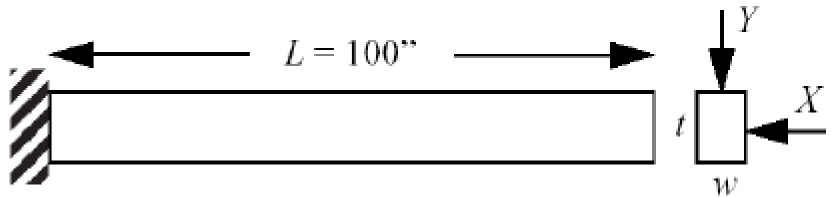
Sensitivity Analysis References

- Saltelli A., Ratto M., Andres T., Campolongo, F., et al., Global Sensitivity Analysis: The Primer, Wiley, 2008.
- J. C. Helton and F. J. Davis. Sampling-based methods for uncertainty and sensitivity analysis. Technical Report SAND99-2240, Sandia National Laboratories, Albuquerque, NM, 2000.
- Sacks, J., Welch, W.J., Mitchell, T.J., and Wynn, H.P. Design and analysis of computer experiments. Statistical Science 1989; 4:409–435.
- Oakley, J. and O'Hagan, A. Probabilistic sensitivity analysis of complex models: a Bayesian approach. J Royal Stat Soc B 2004; 66:751–769.

- Dakota User's Manual
  - Parameter Study Capabilities
  - Design of Experiments Capabilities/Sensitivity Analysis
  - Uncertainty Quantification Capabilities (for MC/LHS sampling)
- Corresponding Reference Manual sections

BACKUP SA SLIDES

Cantilever Beam Analysis Problem



- Example sensitivity analysis goals:
  - Determine influence of beam_width, beam_thickness, R (yield stress), E (Young's modulus), X (horizontal load), Y (vertical load) on each of area (weight), stress, and displacement
  - Determine whether these have only a main effect or if parameter interactions and higher order effects figure in

weight (*area = w*t*)

$$stress = \frac{600}{wt^2} Y + \frac{600}{w^2 t} X \leq R$$

$$displacement = \frac{4L^3}{Ewt} \sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{X}{w^2}\right)^2} \leq D_0$$

Given values of *w, t, R, E, X, Y,* Dakota's `mod_cantilever` driver computes *area, stress-R, displacement-D_0*

Optional: Additional Sensitivity Analysis Capabilities

- Variance-based decomposition (via sampling or PCE)
  - Goal: Apportion uncertainty in responses to uncertainty in inputs
  - Expensive: K*(N+2) simulations required, K = # samples, N = # variables, recommended K ≥ 100
  - Exercise: Modify the sensitivity analysis method to perform variance-based decomposition on the cantilever problem

- Main Effects/Analysis of Variance (ANOVA)
  - Goal: Determine effect of a variable on mean behavior
  - Uses design of experiments: Coverage of space (e.g., space filling, interior, boundaries/extremes, etc.) varies by design
  - Exercise: Modify the sensitivity analysis method to perform a main effects analysis using an orthogonal array on the cantilever problem

Results for VBD and Main Effects

Global sensitivity indices for each response function:			
weight	**Sobol**	**indices:**	
	Main	**Total**	
	0.00	0.00	**R**
	0.00	0.00	**E**
	0.00	0.00	**X**
	0.00	0.00	**Y**
	0.49	0.51	**beam_width**
	0.51	0.52	**beam_thickness**
stress	**Sobol**	**indices:**	
	Main	**Total**	
	0.16	0.13	**R**
	0.00	0.00	**E**
	0.37	0.36	**X**
	0.39	0.36	**Y**
	0.08	0.08	**beam_width**
	0.11	0.12	**beam_thickness**
displ	**Sobol**	**indices:**	
	Main	**Total**	
	0.00	0.00	**R**
	0.90	0.92	**E**
	0.02	0.02	**X**
	0.07	0.08	**Y**
	0.02	0.01	**beam_width**
	0.04	0.05	**beam_thickness**

Variance-based decomposition

Response Function 1

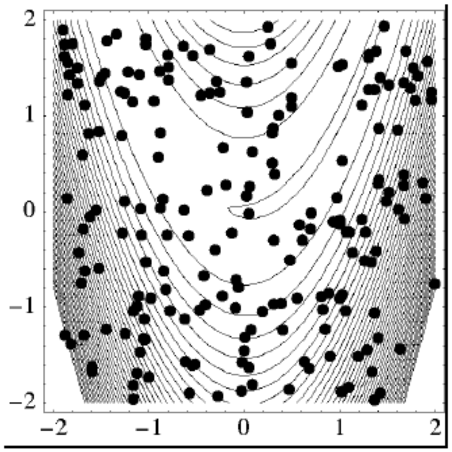
ANOVA	Table	for	Factor	(Variable)	4	
Source	of	Sum	of	Mean	Sum	
Variation	DoF	Squares	of	Squares	Fdata	
Between	Groups	22	2.18E-03	9.89E-05	**3.22E-03**	**Y**
Within	Groups	506	1.55E+01	3.07E-02		
Total	528	1.55E+01				
ANOVA	Table	for	Factor	(Variable)	5	
Source	of	Sum	of	Mean	Sum	
Variation	DoF	Squares	of	Squares	Fdata	
Between	Groups	22	7.80E+00	3.55E-01	**2.32E+01**	**Beam Width**
Within	Groups	506	7.73E+00	1.53E-02		
Total	528	1.55E+01				
ANOVA	Table	for	Factor	(Variable)	6	
Source	of	Sum	of	Mean	Sum	
Variation	DoF	Squares	of	Squares	Fdata	
Between	Groups	22	7.70E+00	3.50E-01	**2.26E+01**	**Beam Thickness**
Within	Groups	506	7.84E+00	1.55E-02		
Total	528	1.55E+01				

Main Effects Analysis

Same relative ranking across methods.

Sensitivity Analysis with Sampling

- Assume inputs fall within lower and upper bounds
- Generate uniform random samples over these intervals
- Compute response value at each sample point
- Look at correlation results
  - Simple and partial correlations
  - Raw and rank correlations
- *Caution: measures the strength and direction of a linear relationship between variables correlation only*
- Correlation coefficient near
  - 0 indicates no relationship
  - 1 indicates strong positive relationship (as x increases, y increases)
  - -1 indicates strong negative relationship (as x increases, y decreases)



Additional Sensitivity Analysis Capabilities

- Variance-based decomposition
  - Goal: Apportion uncertainty in responses to uncertainty in inputs
  - Expensive: K*(N+2) simulations required, K = # samples, N = # variables, recommended K ≥ 100
  - Exercise: Modify the sensitivity analysis method to perform variance-based decomposition on the “textbook” problem

- Main Effects/Analysis of Variance (ANOVA)
  - Goal: Determine effect of a variable on mean behavior
  - Uses design of experiments: Coverage of space (e.g., space filling, interior, boundaries/extremes, etc.) varies by design
  - Exercise: Modify the sensitivity analysis method to perform a main effects analysis using an orthogonal array on the “textbook” problem

Design of Experiments

- Design of Experiments (DOE) is sometimes used to help understand variable importance.
- Design and Analysis of Computer Experiments (DACE) refers to DOE for computer models.
- Big difference between physical and computer experiments: many of our codes are deterministic (e.g., same input settings will produce same outputs under replication), whereas physical experiments are usually not.
- DACE can be used to help understand range of outputs and important variables. It is generally NOT an uncertainty propagation method.
- Prototypical method: orthogonal arrays

Orthogonal Arrays

- For each level of one factor, all levels of the other factors occur an equal number of times: "cancel out" effect.
- Orthogonality: statistical independence between the columns of the experimental design matrix
- Standard analysis involves comparison of main effects: Is the mean of factor 1 at level 1 different than the mean of factor 1 at level 2?
- Large databases of OAs have been compiled by various industry and statistical organizations.
- Example:

Exp. No	Var. 1	Var. 2	Var. 3	Var. 4	Var. 5	Var. 6	Var. 7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Textbook Exercise:

SA with Sampling

```
method
 nond_sampling
  sample_type
   lhs
  samples = 100
  seed = 12345

variables
 uniform_uncertain = 2
  lower_bounds = -1 -1
  upper_bounds = 3 3
  descriptors = 'x1' 'x2'

interface
 analysis_drivers = 'text_book'
  direct

responses
 num_response_functions = 1
 no_gradients
 no_hessians
```

```
Statistics based on 100 samples:

Moments for each response function:
response_fn_1:  Mean = 6.3982749478e+00     Std. Dev. =
6.0079987768e+00
                Coeff. of Variation = 9.3900290716e-01

95% confidence intervals for each response function:
response_fn_1:  Mean = ( 5.2061576460e+00, 7.5903922496e+00 )
                Std Dev = ( 5.2750640532e+00, 6.9793435129e+00 )

Simple Correlation Matrix among all inputs and outputs:
                    x1            x2           response_fn_1
           x1   1.00000e+00
           x2  -6.24801e-03   1.00000e+00
response_fn_1   1.40254e-01   4.25038e-02   1.00000e+00

Partial Correlation Matrix between input and output:
           response_fn_1
     x1   1.40649e-01
     x2   4.38140e-02

Simple Rank Correlation Matrix among all inputs and outputs:
                    x1            x2           response_fn_1
           x1   1.00000e+00
           x2  -3.70837e-03   1.00000e+00
response_fn_1   9.11371e-02  -2.59226e-02   1.00000e+00

Partial Rank Correlation Matrix between input and output:
           response_fn_1
     x1   9.10722e-02
     x2  -2.56917e-02
```

Sampling Input

Sampling Output

Textbook Exercise:
SA with Sampling, Tabular Data

```
method
 nond_sampling
  sample_type
   lhs
  samples = 100
  seed = 12345

variables
 uniform_uncertain = 2
  lower_bounds = -1 -1
  upper_bounds = 3 3
  descriptors = 'x1' 'x2'

interface
 analysis_drivers = 'text_book'
  direct

responses
 num_response_functions = 1
 no_gradients
 no_hessians

strategy
 tabular_graphics_data
  tabular_graphics_file = 'training_sa.dat'
 single_method
```

%eval_id	x1	x2	response_fn_1
1	2.232734001	2.875924025	14.69328591
2	1.793275972	2.311273484	3.352469388
3	0.9651725828	1.92367612	0.727913444
4	1.547939246	-0.7228731781	8.900899832
5	2.528758002	-0.239438414	7.821974951
6	1.90385464	0.4789291991	0.7411328243
7	-0.6187168833	0.6355368689	6.883325151
8	1.596148902	0.5453621751	0.1690276001
9	2.443838741	2.394096379	8.123060879
10	2.7503716	2.732582642	18.39793401
11	1.51201481	-0.5424977029	5.729791937
12	2.665546472	1.466315933	7.742610686
13	0.4785859039	2.577518705	6.266871826
14	0.7086981446	1.228698622	0.009936281628
15	-0.9073449776	0.3829414625	13.37976747
16	2.392407576	0.0318674671	4.637435939
17	1.029187563	0.6998165904	0.008120552171
18	-0.5761782119	0.3590574848	6.340696728
19	0.9375857843	2.132517316	1.645066316
20	-0.7518175888	1.259806897	9.422487756

Sampling Input

Sampling Output

Textbook Exercise:

SA with Sampling, Different Bounds

Sandia National Laboratories

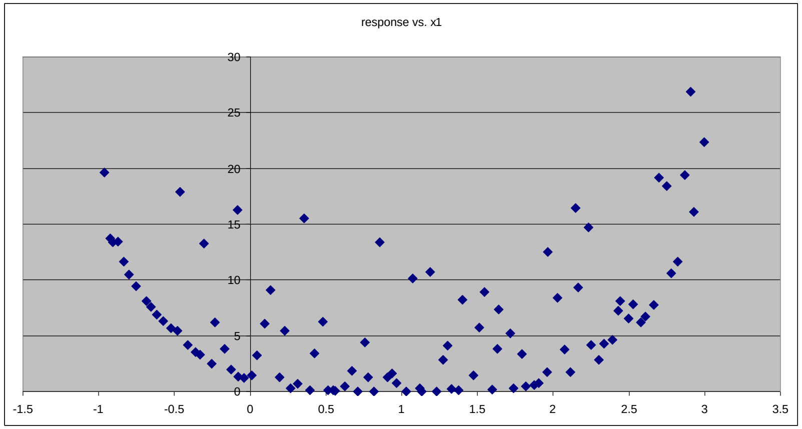
```
method
 nond_sampling
  sample_type
   lhs
  samples = 100
  seed = 12345

variables
 uniform_uncertain = 2
  lower_bounds = 1 1
  upper_bounds = 3 3
  descriptors = 'x1' 'x2'

interface
 analysis_drivers = 'text_book'
  direct

responses
 num_response_functions = 1
 no_gradients
 no_hessians

strategy
 tabular_graphics_data
  tabular_graphics_file = 'training_sa.dat'
 single_method
```

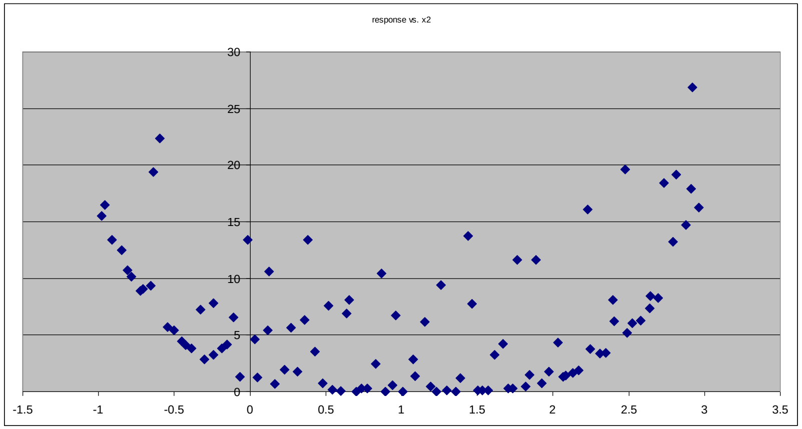
```
Statistics based on 100 samples:

Moments for each response function:
response_fn_1:  Mean = 6.4041210434e+00     Std. Dev. =
6.4508645208e+00
                          Coeff. of Variation = 1.0072989684e+00

95% confidence intervals for each response function:
response_fn_1:  Mean = ( 5.1241295700e+00, 7.6841125169e+00 )
                          Std Dev = ( 5.6639032080e+00, 7.4938096891e+00 )

Simple Correlation Matrix among all inputs and outputs:
                          x1               x2            response_fn_1
                x1   1.00000e+00
                x2  -6.24801e-03   1.00000e+00
response_fn_1   6.40056e-01   5.95030e-01   1.00000e+00

Partial Correlation Matrix between input and output:
               response_fn_1
          x1   8.01025e-01
          x2   7.79668e-01

Simple Rank Correlation Matrix among all inputs and outputs:
                          x1               x2            response_fn_1
                x1   1.00000e+00
                x2  -3.70837e-03   1.00000e+00
response_fn_1   6.77288e-01   6.06673e-01   1.00000e+00

Partial Rank Correlation Matrix between input and output:
               response_fn_1
          x1   8.54822e-01
          x2   8.28019e-01
```

Sampling Input | Sampling Output

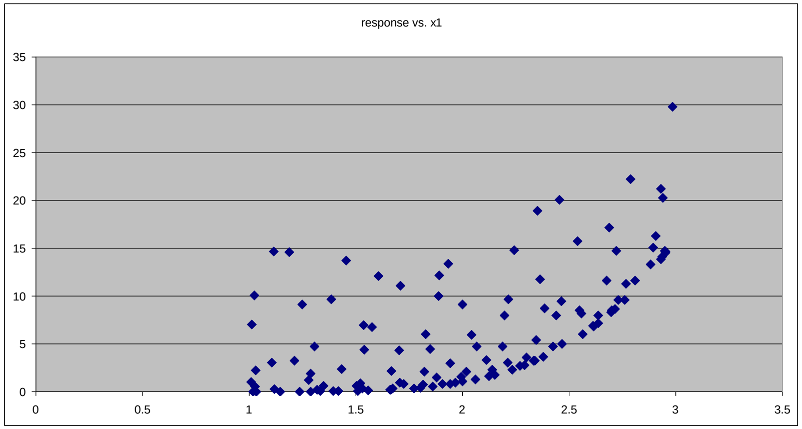
Textbook Exercise:
SA with Sampling, Scatter Plots

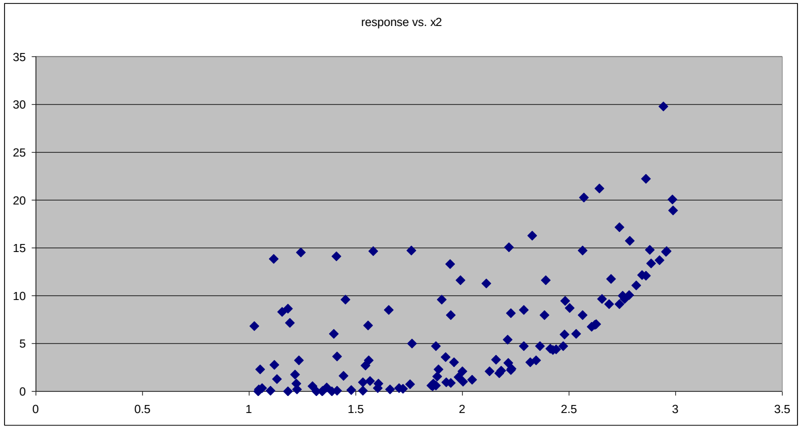
Bounds = [-1, 3]





Bounds = [1, 3]





Textbook Exercise:
Variance-Based Decomposition

```
method
 nond_sampling
  sample_type
   lhs
  variance_based_decomp
  samples = 100
  seed = 12345

variables
 uniform_uncertain = 2
  lower_bounds = 1 1
  upper_bounds = 3 3
  descriptors = 'x1' 'x2'

interface
 analysis_drivers = 'text_book'
  direct

responses
 num_response_functions = 1
 no_gradients
 no_hessians

strategy
 tabular_graphics_data
  tabular_graphics_file = 'training_sa.dat'
 single_method
```

```
Variance Based Decomposition Sensitivity Indices
These indices measure the importance of the uncertain input
variables in determining the uncertainty (variance) of the output.
Si measures the main effect for variable i itself, while Ti
measures the total effect (including the interaction effects
of variable i with other uncertain variables.)

response_fn_1
          x1:  Si = 4.92001e-01  Ti = 5.81994e-01
          x2:  Si = 5.18284e-01  Ti = 5.34544e-01
<<<<< Function evaluation summary: 400 total (400 new, 0 duplicate)
```

Sampling Input

Sampling Output

Textbook Exercise:
Main Effects/ANOVA

Sandia National Laboratories

```
method
 dace
  oas
  main_effects
  samples = 100

variables
 uniform_uncertain = 2
  lower_bounds = 1 1
  upper_bounds = 3 3
  descriptors = 'x1' 'x2'

interface
 analysis_drivers = 'text_book'
  direct

responses
 num_response_functions = 1
 no_gradients
 no_hessians

strategy
 tabular_graphics_data
  tabular_graphics_file = 'training_sa.dat'
 single_method
```

```
Warning: For orthogonal array sampling, the number of samples should be an
         integer multiple of (num_symbols)^2, and num_symbols should be
         either 4 or a prime number.

Adjusting the number of symbols and samples....
  num_variables   = 2
  OLD num_samples = 100    OLD num_symbols = 0
  NEW num_samples = 121    NEW num_symbols = 11

DACE method = oas Samples = 121 Symbols = 11 Seed (system-generated) =
201266

---------------------------------------
Main effects for  response_fn_1:
---------------------------------------
ANOVA Table for Factor (Variable) 1
Source of                    Sum of        Mean Sum
Variation          DoF      Squares       of Squares       Fdata        p-value
Between Groups      10   2.25672e+03   2.25672e+02   1.08953e+01   ************
Within Groups      110   2.27840e+03   2.07128e+01
Total              120   4.53512e+03

ANOVA Table for Factor (Variable) 2
Source of                    Sum of        Mean Sum
Variation          DoF      Squares       of Squares       Fdata        p-value
Between Groups      10   2.21493e+03   2.21493e+02   1.05010e+01   ************
Within Groups      110   2.32019e+03   2.10926e+01
Total              120   4.53512e+03
```

Sampling Input

Sampling Output